Conference Handbook

2018 Australia-China joint conference on Geometric Analysis and Differential Geometry

Time: 13-17 August 2018

Place: the hotel Oaks Calypso Plaza, Coolangatta, Gold Coast
https://www.minorhotels.com/en/oaks/oaks-calypso-plaza

Organized by the University of Queensland

***in collaboration with the Beijing International Center for Mathematics Research*.**

Sponsors:

The Australian National University,
The University of Queensland,
The University of Sydney,
The University of Wollongong.

Scientific Committee

- **Ben Andrews – The Australian National University, Canberra**
- **Gang Tian - The Beijing International Center for Mathematics Research, Beijing, China**
- **Neil Trudinger - The Australian National University, Canberra.**
- **Zhou Zhang - The University of Sydney, Sydney**

Organizing Committee

- **Min-Chun Hong (Chair) - The University of Queensland, Brisbane**
- **Donglin Li- The Beijing International Center for Mathematics Research**
- **Joe Grotowski - The University of Queensland, Brisbane**

Schedule

Monday 13 August

Time	Speakers	Title	Chair
9:00-9:50	Gang Tian	V-solitons and Ricci Flow	Min-Chun Hong
10:00-10:30	Tea break		
10:30-11:20	Neil Trudinger	Open problems in elliptic PDE	Min-Chun Hong
11:30-12:20	Xingwang Xu	Existence results for the Einstein-scalar field Lichnerowicz equations	Min-Chun Hong
12:30-14:00	Lunch		
14:00-14:50	Valentina Wheeler	On Chen submanifolds and the Chen flow	Neil Trudinger
15:00-15:50	Weimin Sheng	Convergence of The CR Yamabe Flow	Neil Trudinger
16:00-16:30	Tea break		
16:30-17:20	Feng Wang	The existence of Kahler-Einstein metrics on K-polystable Q-Fano varieties with non-positive discrepancies	Neil Trudinger

Tuesday 14 August

Time	Speaker	Title	Chair
9:00-9:50	Weiping Zhang	From Bergman kernel to analytic torsion	Gang Tian
10:00-10:30	Tea break		
10:30-11:20	Gaven Martin	Teichm\”uller Theory and the calculus of variations.	Gang Tian
11:30-12:20	Jixiang Fu	Some positivity results in Kaehler geometry.	Gang Tian
12:30-14:00	Lunch		
14:00-14:50	Meng Wang	A note on the existence of the fractional Yamabe problem	Weiping Zhang
15:00-15:50	Zhenlei Zhang	Relative volume comparison of Ricci flow and its application	Weiping Zhang
16:00-16:30	Tea break		
16:30-17:20	Haotian Wu	Asymptotic behaviours of some Ricci flow solutions	Weiping Zhang
17:30-18:00	Conference dinner at 6PM		

Wednesday 15 August

Time	Speaker	Title	Chair
9:00-9:50	Jiayu Li	Compactness and singularity related to harmonic maps	Joe Grotowski
10:00-10:30	Tea break		
10:30-11:20	Ben Andrews	Hypersurface flows and geometric inequalities in hyperbolic space	Joe Grotowski
11:30-12:20	Haizhong Li	Inverse mean curvature flow and some geometric applications	Joe Grotowski
12:30-14:00	Lunch		
14:00-14:50	Free discussion		
15:00-15:50	Free discussion		
16:00-16:30	Free discussion		
16:30-17:20	Free discussion		
17:30-18:00			

Thursday 16 August

Time	Speaker	Title	Chair
9:00-9:50	Jie Qing	Compactness and uniqueness of asymptotically hyperbolic Einstein manifolds	Ben Andrews
10:00-10:30	Tea break		
10:30-11:20	Rod Gover	Singular Yamabe metrics and hypersurface curvatures	Ben Andrews
11:30-12:20	Xi Zhang	The Hermitian Yang-Mills flow	Ben Andrews
12:30-14:00	Lunch		
14:00-14:50	Zhou Zhang	Kahler-Ricci Flows with Infinite Time Singularities	Jiayu Li
15:00-15:50	Yu Zheng	The convexity of algebraic curvature operator and Harnack inequalities along the Ricci flow	Jiayu Li
16:00-16:30	Tea break		
16:30-17:20	Changwei Xiong	Pointwise gradient estimates via two-point functions for elliptic equations on manifolds	Jiayu Li
17:30-18:00			

Friday 17 August

Time	Speaker	Title	Chair
9:00-9:50	Xiaohua Zhu	Steady Ricci Solitons with positive curvature	Zhou Zhang
10:00-10:30	Tea break		
10:30-11:20	James McCoy	Contraction of convex hypersurfaces by non-smooth concave functions of their curvature	Zhou Zhang
11:30-12:20	Xiao Zhang	The positive energy theorem for asymptotically anti-de Sitter spacetimes with distributional curvature	Zhou Zhang
12:30-14:00	Lunch		
14:00-14:50	Jiakun Liu	A boundary value problem for Monge-Ampere equations.	Zhou Zhang
15:00-15:50	Tea break		
16:00-16:30	Free discussion		
16:30-17:20	Free discussion		
17:30-17:00	Conference end		

Title and Abstract

Speaker: Benjamin Andrews (The Australian National University)

Title: Hypersurface flows and geometric inequalities in hyperbolic space

Abstract: In this talk I will describe some recent work (joint with Yong Wei (ANU) and Xuzhong Chen (Hunan)) on the application of curvature-driven evolution equations for hypersurfaces to prove geometric inequalities in hyperbolic space. I will first discuss some earlier results which give relations between integrals of elementary symmetric functions of curvature for horospherically convex hypersurfaces. Then I will discuss two generalisations: First, we extend these inequalities under the weaker condition of positive intrinsic curvature; and second, we show that the horospherically convex condition in fact admits a sharper family of inequalities. If time permits, I will also discuss an intriguing relation between flows of horospherically convex hypersurfaces and conformal deformation of metrics on the sphere.

Speaker: Jixiang Fu (Fudan University, China)

Title: Some positivity results in Kaehler geometry.

Abstract: I will talk about the relations of the balanced cone of a compact Kaehler manifold with the Kaehler cone, the pseudo-effective cone, and the movable cone. I will also talk about Teissier's proportionality theorem on a compact Kaehler manifold and give the sketch of its proof.
This talk is based on my joint work with Jian Xiao.

Speaker: Rod Gover (Auckland, New Zealand)

Title: Singular Yamabe metrics and hypersurface curvatures

Abstract: For metrics that are singular in the sense of conformal compactification, the scalar curvature extends smoothly to points ``at infinity'' of the given n-manifold. This observation is linked to interesting variants of the Yamabe equation, solutions of which make constant a generalised scalar curvature. This curvature agrees with the usual scalar curvature at all points where the metric is defined but is also defined where the metric is singular. In the case that this curvature is non-zero, smooth solutions have the interesting property that the metric singularity set, if non-empty, is a smoothly embedded hypersurface that satisfies the Willmore equation (in the case of dimension 3) or higher order conformally invariant equations that, in the case of n-odd, are higher dimensional analogues of the Willmore equation. This leads to some interesting questions that generalise parts of the original Obata conjecture. Another curvature is linked to the problem in that the energies for these

equations can be recovered from a Q-curvature defined along the hypersurface. This is joint work with Andrew Waldron

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Speaker: Jiayu Li (University of Science and Technology of China)

Title: Compactness and singularity related to harmonic maps

Abstract: In the talk I will review our recent works on compactness and regularity related to harmonic maps, especially on triholomorphic maps between hyperkaehler manifolds.

Speaker: Haizhong Li (Tsinghua University, China)

Title: Inverse mean curvature flow and some geometric applications

Abstract: In this talk, we give some important properties of inverse mean curvature flow for hypersurfaces with nonnegative sectional curvature in a hyperbolic space. By use of the properties of inverse mean curvature flow, we prove some geometric inequalities for such hypersurfaces. This is a joint work with Dr Yingxiang Hu.

Speaker: Jiakun Liu (The University of Wollongong)

Title: A boundary value problem for Monge-Ampere equations.

Abstract: In this talk, we will present a recent result on the global $C^{2,\alpha}$ and $W^{2,p}$ regularity for the Monge-Ampere equation subject to a natural boundary condition arising in optimal transportation. This is a joint work with Shibing Chen and Xu-Jia Wang.

Speaker: Gaven Martin (New Zealand Institute for Advanced Study)

Title: Teichm\"uller Theory and the calculus of variations.

Abstract: Classical Teichm\"uller theory is not variational and the corresponding lack of Euler-Lagrange type equations is a drawback in many applications and in discussions of associated metrics. Ahlfors actually gave the first rigorous proof of Teichm\"ullers theorem based on a variational approach and left many questions open. Recent advances allow us to close the loop on this approach the construction of an L^p-Teichm\"uller theory. We will see that as $p\tp\infty$ we get the classical theory, and (surprisingly) as $p\to 1$ we get the harmonic mapping theory. Thus these theories interpolate between two classical approaches. In particular we study minimisers of the p-conformal energy functionals,
\[{\cal E}_p[f] = \iint_\ID \IK^p(z,f) \; dz, \hskip15pt f|\IS=f_0 \]
for Sobolev self homeomorphisms of the unit disk $\ID$ with prescribed boundary values $f_0:\IS\to\IS$ and pointwise distortion function $\IK=\IK(z,f)=\|Df(z)\|^2/det Df(z)$. We establish that minimisers are smooth

diffeomorphisms via a novel approach to the improved regularity of nonlinear distributional equations.

Speaker: James McCoy (The University of Newcastle)

Title: Contraction of convex hypersurfaces by non-smooth concave functions of their curvature

Abstract: In previous work, the authors with collaborators considered contraction of convex hypersurfaces by degree one homogeneous non-smooth convex functions of their principal curvatures, showing via approximating the speed and obtaining uniform estimates that solution hypersurfaces contract to round points in finite time, with the convergence under suitable rescaling exponential in C^2 to the sphere. The first two authors obtained similar results for evolving surfaces, without any convexity condition on the speed, using a different mollification of the speed. In this talk we show that same mollification may also be used in the case of non-smooth concave speeds. Again the evolving hypersurfaces shrink in finite time to round points, where the convergence under rescaling to the sphere is exponential in C^2. We characterise the extinction point as round using a Stampacchia iteration procedure.

Speaker: Jie Qing (Peking University, China)

Title: Compactness and uniqueness of asymptotically hyperbolic Einstein manifolds

Abstract: In this talk I would like to report on some recent progress on compactness and uniqueness of asymptotically hyperbolic Einstein manifolds. I will start with my joint work with Gang Li and Yuguang Shi on the lower bound of the volume growth and interior curvature estimates. Then I will report some my recent joint work with Alice Chang and Yuxin Ge. If time permits, I will also talk some my recent joint work with Xiaoshang Jin on compactness and uniqueness in general dimensions.

Speaker: Weimin Sheng (Zhejiang University, China)

The title: Convergence of The CR Yamabe Flow

Abstract: In this talk, I will report my recent work on CR Yamabe flow. We study the CR Yamabe flow on a compact strictly pseudoconvex CR manifold M of real dimension $2n+1$. We prove convergence of the CR Yamabe flow when $n=1$ or M is spherical and the CR Yamabe invariant is positive. This is my joint work with Pak Tung Ho and Kunbo Wang.

Speaker: Gang Tian (Peking University, China)

Title: V-solitons and Ricci Flow

Abstract: This is an expository talk on the V-soliton equation which was introduced by Lanave and myself in order to study finite-time singularity of Kähler-Ricci flow. I will discuss how it is related to Ricci Flow and some recent progresses. I will also show some examples**.**

Speaker: Neil Trudinger (The Australian National University)

Title: Open problems in elliptic PDE.

Abstract: We present some open problems involving elliptic partial differential operators which are also related to Chinese and Australian mathematical research.

Speaker: Feng Wang (Zhejiang University, China)

Title: The existence of Kahler-Einstein metrics on K-polystable Q-Fano varieties with non-positive discrepancies

Abstract: I will talk out the recent work with Professor Tian and Chi Li. At first, we extend Tian's solution of YTD conjecture to the log smooth case. Then for a K-polystable Q-Fano varieties X with non-positive discrepancies, we show that there exists conic KE metrics on the resolution and these metrics converges to the singular KE metric on X.

Speaker: Meng Wang (Zhejiang University, China)

The title: A note on the existence of the fractional Yamabe problem

Abstract: We prove some existence results for the fractional Yamabe problem in the case that the boundary manifold is umbilic.

Speaker: Valentina Wheeler (The University of Wollongong)

Title: On Chen submanifolds and the Chen flow

Abstract: Chen's operator for a submanifold is the twice iterated Laplacian on the pullback bundle, sometimes known as the rough Laplacian in the literature. Chen's conjecture is that if Chen's operator applied to the immersion map vanishes, then the submanifold is minimal. In the last few years, work has progressed on the parabolic flow with velocity corresponding to Chen's operator applied to the immersion. This parabolic flow has an

interesting variational characterisation, being between the biharmonic map heat flow and the Willmore flow. Algebraically, the Chen flow sits instead between surface diffusion and Willmore flow. Qualitatively its behaviour is much closer to the mean curvature flow. In particular, spheres shrink to points in finite time. In this talk we describe the Chen operator, Chen's conjecture, and some recent work on Chen's flow in two and four dimensions.
This is joint work with Yann Bernard and Glen Wheeler.

Speaker: Haotian Wu (The University of Sydney)

Title: Asymptotic behaviours of some Ricci flow solutions

Abstract: We survey recent development concerning the asymptotic behaviours, particularly those of singularity formation and of dynamical stability, of some Ricci flow solutions.

Speaker: Changwei Xiong (The Australian National University)

Title: Pointwise gradient estimates via two-point functions for elliptic equations on manifolds

Abstract: Pointwise gradient estimates for bounded solutions of elliptic equations on Euclidean space have attracted much attention since the study by L. Modica in 1985. Most of them are known to be derived by the P-function method, a method which seems to work only for equations of special form. In this talk we will present how to use the two-point functions method to obtain these pointwise gradient estimates. Moreover, it turns out that the method by two-point functions can apply well to a much wider class of elliptic equations on Euclidean space, and also to the problems on Riemannian manifolds and Finsler manifolds with suitable lower bounds on (weighted) Ricci curvature. This is a joint work with Ben Andrews.

Speaker: Xingwang Xu (Nanjing University, China)

Title: Existence results for the Einstein-scalar field Lichnerowicz equations

Abstract: In this talk, I shall discuss some new existence and multiplicity results for positive solutions to the Einstein-scalar
field Lichnerowicz equation on closed manifolds. This equation arises from the Hamiltonian constraint equation for the Einstein-scalar field system in general relativity. The some case in the results, including the method, was conjectured by Choquet-Bruhat, Isenberg, and Pollack. The talk is based on several joint work with Dr. Ngo Quoc Anh.

Speaker: Weiping Zhang (Nankai University, China)

Title: From Bergman kernel to analytic torsion

Abstract: We describe a joint work with Jean-Michel Bismut and Xiaonan Ma on asymptotics of the Ray-Singer analytic torsion associated to flat vector bundles arising from the Kodaira vanishing property.

Speaker: Xi Zhang (University of Science and Technology of China)

Title: The Hermitian Yang-Mills flow

Abstract: In this talk, we first recall some classical results on the differential geometry of holomorphic vector bundles, and introduce our recent work on the existence of canonical metrics, Bogomolov type inequalities and the Hermitian Yang-Mills flow. These works are joint with Jiayu Li, Yanci Nie, Chuanjing Zhang and Pan Zhang.

Speaker: Xiao Zhang (Academy Sinica, China)

Title: The positive energy theorem for asymptotically anti-de Sitter spacetimes with distributional curvature

Abstract: The positive energy theorem for asymptotically anti-de Sitter spacetimes with C^{2,\alpha} metrics was proved by Wang-Xie-Zhang earlier, which involves ten conserved quantities of total energy-momentum. However, when the metrics are only C^0 and W^{1,2} and the curvatures are in the sense of distribution, there are some essential difficulties to define the ten total energy-momenta. In this talk, we will provide the corresponding positive energy theorem in this case.
The talk is based on the joint work Yaohua Wang.

Speaker: Zhenlei Zhang (Capital Normal University, China)

Title: Relative volume comparison of Ricci flow and its application

Abstract: In the talk we will present a relative volume comparison of Ricci flow. It is a refinement of Perelman's no local collapsing theorem and can be viewed as an analogy of Bishop-Gromov volume comparison. It can be applied to Ricci flow with a collapsing structure. One application to the study of Gromov-Hausdorff convergence of Kahler-Ricci flow on smooth minimal models will also be discussed. The talk is based on a joint work with Professor Tian.

Speaker: Zhou Zhang (The University of Sydney)

Title: Kahler-Ricci Flows with Infinite Time Singularities

Abstract: We survey the study of Kahler-Ricci flow with infinite time singularities. The requirement of infinite time puts more restrictions on the underlying manifold than the finite time case, and people can aim at obtaining the canonical metric, generalised Kahler-Einstein metric, as the limit. Although there is no weak convergence along the flow in general, we do have more understanding on the geometry for interesting cases.

Speaker: Yu Zheng (East China Normal University, China)

Title: The convexity of algebraic curvature operator and Harnack inequalities along the Ricci flow

Abstract: In this talk, we will first introduce our recent year's study on one new called λ positive algebraic curvature operator preserving along the Ricci flow. It should be the generalization on the positive or 2-positive curvature operator by R. Hamilton and B. Wilking so on respectively. Then the applications on the Differential Harnack inequalities and some open problems will be discussed. It's the jointed works with Xiaodong Cao.

Speaker: Xiaohua Zhu (Peking University, China)
Title: Steady Ricci Solitons with positive curvature

Abstract: I will talk about some recent results on classification of steady Ricci Solitons with positive sectional curvature, in particular, we prove such Ricci Solitons must be rotational if the curvature has linear decay. Moreover, in the case of four dimension, the positive sectional curvature can be weakened by the positive Ricci curvature.
